Chapter 5 Review Exercise

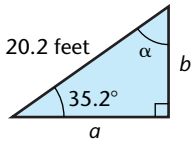
Work through all the problems in this chapter review and check answers in the back of the book. Answers to all review problems are there, and following each answer is a number in italics indicating the section in which that type of problem is discussed. Where weaknesses show up, review appropriate sections in the text.

A

1. Find the radian measure of a central angle opposite an arc 15 centimeters long on a circle of radius 6 centimeters.

2. In a circle of radius 3 centimeters, find the length of an arc opposite an angle of 2.5 radians.

3. Solve the triangle:



4. Find the reference angle associated with each angle θ:
(A) $\theta = \pi/3$ (B) $\theta = -120°$
(C) $\theta = -13\pi/6$ (D) $\theta = 210°$

5. In which quadrants is each negative?
(A) $\sin\theta$ (B) $\cos\theta$ (C) $\tan\theta$

6. If $(4, -3)$ is on the terminal side of angle θ, find:
(A) $\sin\theta$ (B) $\sec\theta$ (C) $\cot\theta$

7. Complete Table 1 using exact values. Do not use a calculator.

TABLE 1

θ°	θ rad	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
0°					ND*		
30°							
45°	π/4		$1/\sqrt{2}$				
60°							
90°							
180°							
270°							
360°							

*ND = Not Defined

8. What is the period of each of the following?
(A) $y = \cos x$ (B) $y = \csc x$ (C) $y = \tan x$

9. Indicate the domain and range of each.
(A) $y = \sin x$ (B) $y = \tan x$

10. Sketch a graph of $y = \sin x$, $-2\pi \le x \le 2\pi$.

11. Sketch a graph of $y = \cot x$, $-\pi < x < \pi$.

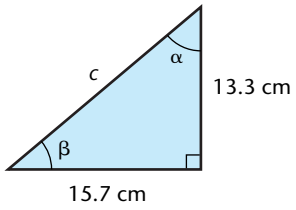
12. Verbally describe the meaning of a central angle in a circle with radian measure 0.5.

13. Describe the smallest shift of the graph of $y = \sin x$ that produces the graph of $y = \cos x$.

B

14. Change 1.37 radians to decimal degrees to two decimal places.

15. Solve the triangle:



16. Indicate whether the angle is a I, II, III, or IV quadrant angle or a quadrantal angle.
(A) $-210°$ (B) $5\pi/2$ (C) 4.2 radians

17. Which of the following angles are coterminal with 120°?
(A) $-240°$ (B) $-7\pi/6$ (C) 840°

18. Which of the following have the same value as cos 3?
(A) cos 3° (B) cos (3 radians) (C) cos $(3 + 2\pi)$

19. For which values of x, $0 \le x < 2\pi$, is each of the following not defined?
(A) $\tan x$ (B) $\cot x$ (C) $\csc x$

20. A circular point $P(a, b)$ moves clockwise around the circumference of a unit circle starting at (1, 0) and stops after covering a distance of 8.305 units. Explain how you would find the coordinates of point P at its final position and how you would determine which quadrant P is in. Find the coordinates of P to three decimal places and the quadrant for the final position of P.

In Problems 21–36, evaluate exactly without the use of a calculator.

21. $\tan 0$ **22.** $\sec 90°$ **23.** $\cos^{-1} 1$

24. $\cos\left(-\frac{3\pi}{4}\right)$ **25.** $\sin^{-1}\frac{\sqrt{2}}{2}$ **26.** $\csc 300°$

27. $\arctan \sqrt{3}$

28. $\sin 570°$

29. $\tan^{-1}(-1)$

30. $\cot\left(-\frac{4\pi}{3}\right)$

31. $\arcsin\left(-\frac{1}{2}\right)$

32. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

33. $\cos(\cos^{-1} 0.33)$

34. $\csc[\tan^{-1}(-1)]$

35. $\sin\left[\arccos\left(-\frac{1}{2}\right)\right]$

36. $\tan\left(\sin^{-1}\frac{-4}{5}\right)$

Evaluate Problems 37–44 to 4 significant digits using a calculator.

37. $\cos 423.7°$

38. $\tan 93°46'17''$

39. $\sec(-2.073)$

40. $\sin^{-1}(-0.8277)$

41. $\arccos(-1.3281)$

42. $\tan^{-1} 75.14$

43. $\csc[\cos^{-1}(-0.4081)]$

44. $\sin^{-1}(\tan 1.345)$

45. Find the exact degree measure of each without a calculator:
(A) $\theta = \sin^{-1}(-1/2)$ (B) $\theta = \arccos(-1/2)$

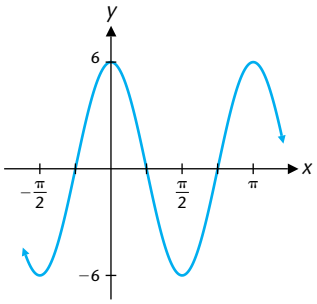
46. Find the degree measure of each to two decimal places using a calculator:
(A) $\Theta = \cos^{-1}(-0.8763)$ (B) $\Theta = \arctan 7.3771$

47. Evaluate $\cos^{-1}[\cos(-2)]$ with a calculator set in radian mode, and explain why this does or does not illustrate the inverse cosine–cosine identity.

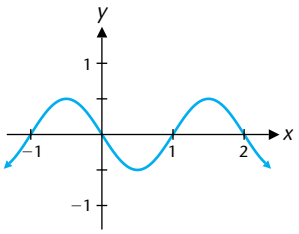
48. Sketch a graph of $y = -2\cos \pi x$, $-1 \le x \le 3$. Indicate amplitude A and period P.

49. Sketch a graph of $y = -2 + 3\sin(x/2)$, $-4\pi \le x \le 4\pi$.

50. Find the equation of the form $y = A\cos Bx$ that has the graph



51. Find the equation of the form $y = A\sin Bx$ that has the graph



52. Describe the smallest shift and/or reflection that transforms the graph of $y = \tan x$ into the graph of $y = \cot x$.

53. Simplify each of the following using appropriate basic identities:

(A) $\sin(-x)\cot(-x)$ (B) $\dfrac{\sin^2 x}{1 - \sin^2 x}$

54. Sketch a graph of $y = 3\sin[(x/2) + (\pi/2)]$ over the interval $-4\pi \le x \le 4\pi$.

55. Indicate the amplitude A, period P, and phase shift for the graph of $y = -2\cos[(\pi/2)x - (\pi/4)]$. Do not graph.

56. Sketch a graph of $y = \cos^{-1} x$, and indicate the domain and range.

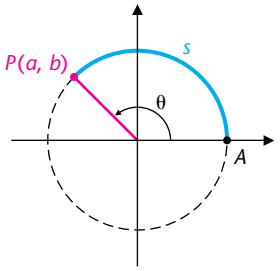
57. Graph $y = 1/(1 + \tan^2 x)$ in a graphing utility that displays at least two full periods of the graph. Find an equation of the form $y = k + A\sin Bx$ or $y = k + A\cos Bx$ that has the same graph.

58. Graph each equation in a graphing utility and find an equation of the form $y = A\tan Bx$ or $y = A\cot Bx$ that has the same graph as the given equation. Select the dimensions of the viewing window so that at least two periods are visible.

(A) $y = \dfrac{2\sin^2 x}{\sin 2x}$ (B) $y = \dfrac{2\cos^2 x}{\sin 2x}$

C

59. If in the figure the coordinates of A are $(8, 0)$ and arc length s is 20 units, find:
(A) The exact radian measure of θ
(B) The coordinates of P to 3 significant digits



60. Find exactly the least positive real number for which:
(A) $\cos x = -\frac{1}{2}$ (B) $\csc x = -\sqrt{2}$

61. Sketch a graph of $y = \sec x$, $-\pi/2 < x < 3\pi/2$.

62. Sketch a graph of $y = \tan^{-1} x$, and indicate the domain and range.

63. Indicate the period P and phase shift for the graph of $y = -5 \tan(\pi x + \pi/2)$. Do not graph.

64. Indicate the period and phase shift for the graph of $y = 3 \csc(x/2 - \pi/4)$. Do not graph.

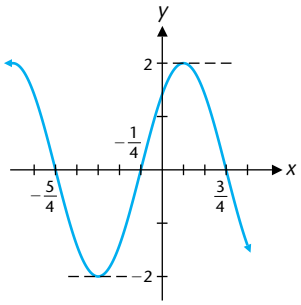
65. Indicate whether each is symmetric with respect to the x axis, y axis, or origin.
(A) Sine (B) Cosine (C) Tangent

66. Write as an algebraic expression in x free of trigonometric or inverse trigonometric functions:

$$\sec(\sin^{-1} x)$$

67. Try to calculate each of the following on your calculator. Explain the results.
(A) $\csc(-\pi)$ (B) $\tan(-3\pi/2)$ (C) $\sin^{-1} 2$

68. The accompanying graph is a graph of an equation of the form $y = A \sin(Bx + C)$, $-1 < -C/B < 0$. Find the equation.



69. Graph $y = 1.2 \sin 2x + 1.6 \cos 2x$ in a graphing utility. (Select the dimensions of the viewing window so that at least two periods are visible.) Find an equation of the form $y = A \sin(Bx + C)$ that has the same graph as the given equation. Find A and B exactly and C to three decimal places. Use the x intercept closest to the origin as the phase shift.

70. A particular wave form is approximated by the first six terms of a Fourier series:

$$y = \frac{4}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{\sin 7x}{7} + \frac{\sin 9x}{9} + \frac{\sin 11x}{11}\right)$$

(A) Graph this equation in a graphing utility for $-3\pi \le x \le 3\pi$ and $-2 \le y \le 2$.
(B) The graph in part A approximates a wave form that is made up entirely of straight-line segments. Sketch by hand the waveform that the Fourier series approximates.

This wave form is called a **pulse wave** or a **square wave,** and is used, for example, to test distortion and to synchronize operations in computers.

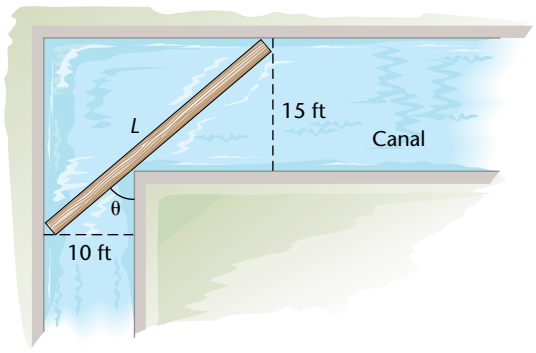
APPLICATIONS

71. Astronomy. A line from the sun to the Earth sweeps out an angle of how many radians in 73 days? Express the answer in terms of π.

★ **72. Geometry.** Find the perimeter of a square inscribed in a circle of radius 5.00 centimeters.

★ **73. Alternating Current.** The current I in alternating electric current has an amplitude of 30 amperes and a period of $\frac{1}{60}$ second. If $I = 30$ amperes when $t = 0$, find an equation of the form $I = A \cos Bt$ that gives the current at any time $t \ge 0$.

74. Restricted Access. A 10-ft-wide canal makes a right turn into a 15-ft-wide canal. Long narrow logs are to be floated through the canal around the right angle turn (see figure). We are interested in finding the longest log that will go around the corner, ignoring the log's diameter.



(A) Express the length of the line L that touches the two outer sides of the canal and the inside corner in terms of θ.
(B) Complete Table 2, each to one decimal place, and estimate from the table the longest log to the nearest foot that can make it around the corner. (The longest log is the shortest distance L.)

TABLE 2

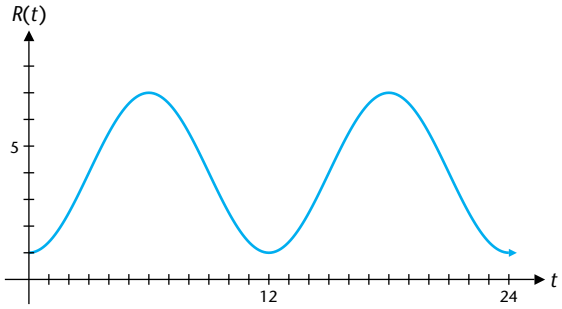
θ (rads)	0.4	0.5	0.6	0.7	0.8	0.9	1.0
L (ft)	42.0						

(C) Graph the function in part A in a graphing utility and use an approximation method to find the shortest distance L to one decimal place, hence, the length of the longest log that can make it around the corner.
(D) Explain what happens to the length L as θ approaches 0 or $\pi/2$.

75. Modeling Seasonal Business Cycles. A soft drink company has revenues from sales over a 2-year period as shown by the accompanying graph, where $R(t)$ is revenue (in millions of dollars) for a month of sales t months after February 1.

(A) Find an equation of the form $R(t) = k + A \cos Bt$ that produces this graph.

(B) Verbally interpret the graph



76. Modeling Temperature Variation. The 30-year average monthly temperature, °F, for each month of the year for Los Angeles is given in Table 3 (*World Almanac*).

(A) Using 1 month as the basic unit of time, enter the data for a 2-year period in your graphing utility and produce a scatter plot in the viewing window. Choose $40 \le y \le 90$ for the viewing window.

(B) It appears that a sine curve of the form

$$y = k + A \sin (Bx + C)$$

will closely model this data. The constants k, A, and B are easily determined from Table 3. To estimate C, visually estimate to one decimal place the smallest positive phase shift from the plot in part A. After determining A, B, k, and C, write the resulting equation. (Your value of C may differ slightly from the answer book.)

(C) Plot the results of parts A and B in the same viewing window. (An improved fit may result by adjusting your value of C slightly.)

TABLE 3

x (mos.)	1	2	3	4	5	6	7	8	9	10	11	12
y (temp.)	58	60	61	63	66	70	74	75	74	70	63	58